Scaling Violations in Yang-Mills Theories and Strings in AdS_5

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Abstract

String solitons in AdS_5 contain information of $\mathcal{N}=4$ SUSY Yang-Mills theories on the boundary. Recent proposals for rotating string solitons reproduce the spectrum for anomalous dimensions of Wilson operators for the boundary theory. There are possible extensions of this duality for lower supersymmetric and even for non-supesymmetric Yang-Mills theories. We explicitly demonstrate that the supersymmetric anomalous dimensions of Wilson operators in $\mathcal{N}=0,1$ Yang-Mills theories behave, for large spin J, at the two-loop level in perturbation theory, like $\log J$. We compile the analytic one- and two-loop results for the $\mathcal{N}=0$ case which is known in the literature, as well as for the $\mathcal{N}=1$ case which seems to be missing.

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1 Introduction

Recent developments in understanding the duality between gravity and gauge interactions points to a new synthesis of ideas about the role of string theory for the infrared behavior of both supesymmetric and non-supersymmetric Yang-Mills theories. These developments came from three different directions.

Firstly the AdS/CFT correspondence was extended to new exact string backgrounds, the so called pp-waves, which are unique and universal limits (Penrose limit) of every space-time which admits null geodesics [1, 2]. In these backgrounds it is frequently possible to solve exactly the quantum spectrum of strings while the correspondence with the conformal field theories on the boundary remains intact. One hopes to get nontrivial information about interesting boundary theories by using more detailed information from the string side[3]-[8]. Penrose limits of backgrounds with various number of supersymmetries constructed as orbifolds and orientifolds have been discussed in [9, 10, 11].

A second development came from the realization that the free string Hamiltonian in AdS_5 background describes a string with spacetime dependent tension, whereby it develops hard components with field theory point-like behavior. The hard component of the AdS_5 strings appears in the energy scaling behavior of the production cross-sections of the process $2 \to n$ strings. This process has been calculated in [12] and found to be similar to the hard scattering processes of QCD. In the language of the old parton model the string in flat space-time is very soft. If viewed as a hadron its average radius diverges logarithmically with the number of partons(wea partons). Interestingly in a AdS_5 background its average radius is finite and calculable around a fixed distance from the boundary of AdS_5[13]. More recently the dual picture for deep inelastic processes has also been studied[14].

The third development concerns the duality between space-time geometry and gauge interactions. In [15], an explicit classical string soliton solution has been found in AdS_5 which represents a collapsed closed string in the form of a rod rotating with constant angular velocity in the equator of S^3 of AdS_5. By considering the deviations from flat space-time for the energy-angular momentum relations for large spin, logarithmic corrections were found similar to the large spin behavior of the anomalous dimensions of Wilson operators for the $\mathcal{N} = 4$ supersymmetric Yang-Mills theories on the boundary. It is interesting to note that similar behavior was found for rotating long strings in the AdS_5 black hole background[16] while rotating strings exhibiting confinement as well as finite-size effects have been studied in [17]. It should be stressed here that for the GKP solution which describes the rotating string in AdS_5, the internal space does not enter anywhere. Thus, the superstring background can be the standard maximally supersymmetric $AdS_5 \times S^5$, as well as any of the available $N = 2$ supergravity backgrounds of the form $AdS_5 \times X^5$ described in [18]. It can even be a non-supersymmetric background with an AdS_5 factor. This seems to indicate that the the large spin behavior of the anomalous dimensions of Wilson operators in Yang-Mills theories is the same (up to possible coefficient differences) for both supersymmetric and non-supersymmetric theories.

In this work we confirm that both $\mathcal{N} = 1$ and $\mathcal{N} = 4$ gauge theories exhibit identical large spin behavior. In particular, we reconsider in detail the existing calculations for anomalous dimensions of Wilson operators in $\mathcal{N} = 0, 1$ supersymmetric Yang-Mills theories up to two loops. We identify the numerical coefficients in front of the confirmed logarithmic behaviour for large spin. Moreover for the case of $\mathcal{N} = 1$ we calculate explicitly the two loop anomalous dimensions from the known results of the non-supersymmetric case[19]. In sect. 2 we recall the GKP rotating strings [15] in

AdS_5 in the limit of large spin (long strings). In sect. 3 we review the formalism of operator product expansion for deep inelastic scattering and we set up our notation. In sect. 4 we exhibit the analytic results of the two loop anomalous dimensions for the $\mathcal{N}=1$ pure Yang-Mills theories. In sect. 5 we give the asymptotic behavior of both supersymmetric and non-supersymmetric cases in the limit of large spin. Finally, in an appendix we compile, for convenience, the long known two loop anomalous dimensions for the non-supersymmetric case(QCD).

2 AdS_5 Rotating Strings in the Large Spin Limit

One of the emerging scenarios, due to Polyakov, provides a further extension to the AdS/CFT correspondence between the AdS_5 supergravity and the boundary theory of $\mathcal{N}=4$ supersymmetric Yang-Mills which possibly may reach out all the way to the non-supersymmetric regime. According to it the AdS_5 space-time which appears as a solution of the quantum non-critical string theory in four dimensions provides a dual description of pointlike field theories. In such a description the fifth dimension plays the role of a non-critical Liouville field[15, 20]. The non-critical string represents the dynamics of gauge field strength lines and it has to live in AdS_5. The AdS_5 radius R satisfies the relation $R^4=\lambda\alpha'^2$ where $\lambda=(g_{YM}^2N)/4\pi$ is the 't Hooft coupling. Weak coupling string interactions correspond to strong gauge interactions on the boundary. As a result, in order to explore the weak gauge coupling regime or equivalently the high energy behavior of the boundary gauge theory, one has to study nonperturbative classical string theory (large N behavior) along with its quantum corrections. The GKP rotating string soliton in AdS_5 offers an intriguing playground for developing and understanding these ideas. In particular, it may provide a useful tool in exploring the transition region between weak and strong gauge couplings or small and big space-time curvatures.

In our present paper we make explicit quantitative comparison between string soliton behavior for large 't Hooft coupling and the analytic results for two loop anomalous dimensions of $\mathcal{N}=0,1$ Yang-Mills theories in the same limit. We begin with the description of the Polyakov string soliton. We follow the parametrization of the global AdS_5 metric

$$ds^2=R^2(-dt^2\cosh^2\rho+d\rho^2+\sinh^2\rho d\Omega_3^2) \tag{2.1}$$

The string soliton rotates at the equator of S^3 and the azimuthal angle depends linearly on time

$$\phi=\omega t \tag{2.2}$$

By choosing the timelike gauge $t=\tau$ and by assuming that the radial coordinate ρ is only a function of σ we obtain the Nambu-Gotto Lagrangian

$$L=-4\frac{R^2}{2\pi\alpha'}\int_0^{\rho_0}d\rho\,\sqrt{\cosh^2\rho-\dot{\phi}^2\sinh^2\rho} \tag{2.3}$$

The maximun radial distance is ρ_0 which is determined by the speed of light

$$\coth^2\rho_0=\omega^2 \tag{2.4}$$

The reparametrization constraints give the equation

$$\rho'^2=e^2(\cosh^2\rho-\omega^2\sinh^2) \tag{2.5}$$

where $\rho' = \frac{d\rho}{d\sigma}$ and e is adjusted so that σ has a period 2π. The space-time energy E and spin J of the rotating string are then given by

$$\begin{aligned} E &= \frac{R^2}{2\pi\alpha'} e \int_0^{2\pi} d\sigma \ \cosh^2\rho \\ S &= \frac{R^2}{2\pi\alpha'} e\omega \int_0^{2\pi} d\sigma \ \sinh^2\rho \end{aligned} \tag{2.6}$$

These expressions determine $E/\sqrt{\lambda}$ and $J/\sqrt{\lambda}$ as functions of ω.

We are interested here in the classical limit $J \gg \sqrt{\lambda}$ which corresponds to ω approaching one from above $\omega = 1 + \eta$ with $0 < \eta \ll 1$. In this limit the end of the string approaches the boundary of the AdS_5 $\rho_0 \to \frac{1}{2}\log(1/\eta)$. By expanding the energy and spin in terms of η we obtain

$$E - J \sim \sqrt{\lambda} \ \log \frac{J}{\sqrt{\lambda}} + \cdots \tag{2.7}$$

On the gauge theory side, $E - J$, i.e, the dimension minus the spin of some composite operator is the twist of the operator. In the case of the leading trajectory (solid rotating string), the leading contributions come from twist-two operators. The deviation from the flat space-time is reminiscent of the behavior of the anomalous dimensions of the twist-two Wilson operators of deep inelastic scattering for large spin. In a sense, the anti-deSitter backround forces the string to develop hard partonic component reminiscent of QCD [12, 13]. In [15] a concrete conjecture about the large J behavior of $E - J$ is made to the effect that the leading term for arbitrary λ will be

$$E - J = f(\lambda) \ \log J \tag{2.8}$$

In [21], the 1−loop corrections were computed

$$f(\lambda) = \frac{\lambda}{\pi} - \frac{(\log 2)}{3\pi}\lambda^2 \tag{2.9}$$

and no higher powers of $\log J$ corrections were found. These calculations hold for large λ but with $J \gg \lambda$. One hopes that it will be possible to analytically continue these results for small values of λ in order to make contact with the perturbative results for the Yang-Mills theories on the boundary. This is the subject of the recent work by Tseytlin et.al in [3].

3 Operator Product expansion in deep-inelastic scatering

In the experiments of deep-inelastic scattering of leptons on nucleons and in the one electroweak gaugeboson exchange approximation, one measures the differential cross sections of the emerging lepton with specific final four-momentum for unpolarized scattering[22, 23]

$$\frac{d\sigma}{d^4 p_l} = l^{\mu\nu} \cdot W_{\mu\nu} \tag{3.1}$$

with

$$l^{\mu\nu} = 4(p_i^\mu p_f^\nu + p_i^\nu p_f^\mu - g_{\mu\nu} p_i \cdot p_f) \tag{3.2}$$

and

$$W_{\mu\nu} = \frac{1}{8\pi}\sum_{\text{spins}}\int d^4x e^{iq\cdot x} < p,s|[J^\mu(x), J^\nu(0)]|p,s> \tag{3.3}$$

In the Bjorken limit $Q^2 = -q^2 \to \infty$ and $x_B = \frac{Q^2}{2p\cdot q}$ fixed where $q = (p_f - p_i)$ is the momentum transfer and p is the nucleus momentum the Fourier transform above is dominated by light-cone distances. The commutator can be calculated as the imaginary part of the time-ordered product of the electroweak currents J^μ

$$\begin{aligned} W_{\mu\nu} &= \frac{1}{2\pi}\mathrm{Im}T_{\mu\nu}(p,q) \\ &= e_{\mu\nu}\frac{1}{2x_B}F_L(x_B,Q^2) + d_{\mu\nu}\frac{1}{2x_B}F_2(x_B,Q^2) + i\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{p\cdot q}F_3(x_B,q^2) \end{aligned} \tag{3.4}$$

where

$$T_{\mu\nu} = i\int d^4x e^{iq\cdot x} < p|T(J^\mu(x)J^\nu(0))|p> \tag{3.5}$$

and F_L, F_2, F_3 are the structure functions which are measured by the experiments (F_3 is relevant for neutrinos). The right-hand side is dominated by the light-cone $x^2 \to 0$ and can be expanded in terms of composite operators multiplied by coefficient functions

$$T_{\mu\nu} = \sum_{N,i}\left(\frac{1}{2x_B}\right)^N \left[e_{\mu\nu}C^N_{L,i}(\frac{Q^2}{\mu^2},\alpha_s) + d_{\mu\nu}C^N_{2,i}(\frac{Q^2}{\mu^2},\alpha_s) + i\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{p\cdot q}C^N_{3,i}(\frac{Q^2}{\mu^2},\alpha_s)\right] A^N_i(\frac{p^2}{\mu^2}) \tag{3.6}$$

where

$$\begin{aligned} e_{\mu\nu} &= g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \\ d_{\mu\nu} &= -g_{\mu\nu} - p_\mu p_\nu\frac{4x_B^2}{q^2} - (p_\mu q_\nu + p_\nu q_\mu)\frac{2x_B}{q^2} \end{aligned} \tag{3.7}$$

$\alpha_s = \frac{g^2_{YM}}{16\pi^2}$ and A^N_i are the matrix elements of the dominant operators O^N_i ($i = \mathrm{NS, quark, gluons}$) beteween the nucleon state. The dominant terms in the light-cone expansion come from lowest twist (dimension minus spin) operators which can be constructed from the Yang Mills theory with fermions. They are the flavour non-singlet (valence quarks) and the singlet ones (sea-quarks and gluons)

$$O^n_{NS,\alpha} = \frac{i^{n-1}}{2n!}\left(\bar{q}\lambda^\alpha\gamma^{\mu_1}D^{\mu_2}\cdots D^{\mu_n}(1\pm\gamma^5)q + \text{permutations-traces}\right) \tag{3.8}$$

where α is a flavour index and the chiral projector appears in the scattering of neutrinos and anti-neutrinos while in the scattering of electrons or positrons is missing. The singlet operators are

$$\begin{aligned} O^n_q &= \frac{i^{n-1}}{n!}(\bar{q}\gamma^{\mu_1}D^{\mu_2}\cdots D^{\mu_n}q + \text{permutations-traces}) \\ O^n_g &= \frac{i^{n-2}}{2n!}\mathrm{Tr}\left(F^{\lambda\mu_1}D^{\mu_2}\cdots D^{\mu_{n-1}}F_\lambda{}^{\mu_n} + \text{permutations-traces}\right) \end{aligned} \tag{3.9}$$

These operators are the twist $2=(n+2)-n$ ones which are dominating in the light-cone expansion. From eq.(3.6) taking moments over the Bjorken variable x_B we project out the spin N term

$$\int_0^1 dx\, x^{N-k} F_i(x,Q^2) = \sum_{i=NS,q,g} C^N_{i,j}(\frac{Q^2}{\mu^2},\alpha_s) A^N_j(\frac{p^2}{\mu^2}) \tag{3.10}$$

where $k=2$ for F_2, F_L and $k=1$ for F_3. The importance of the moment equations (3.10) comes from the renormalization properties of the Wilson operators

$$< p|O^N_i|p >= (p^{\mu_1}\cdots p^{\mu_N} - \text{traces})\, A^N_i(\frac{p^2}{\mu^2}) \tag{3.11}$$

The non-singlet one is multiplicative renormalized

$$O^N_{NS,\text{bare}} = Z^N_{NS} O^N_{NS,\text{ren.}} \tag{3.12}$$

while the singlet ones between physical states mix through a two-by-two matrix renormalization constant

$$O^N_{i,\text{bare}} = Z^N_{ij} O^N_{j,\text{ren.}}\,, \quad i,j=q,g \tag{3.13}$$

Because of these properties, the moment equations give information about the Q^2 evolution of structure functions. Indeed, since the left-hand side is renormalization group invariant, the dependance on the renormalization scale μ of the right-hand side should cancel between the coefficient functions and the operator matrix elements. This gives the renormalization group equations for the coefficient functions

$$\begin{aligned}
\left(\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s} - \gamma^N_{NS}(\alpha_s)\right) C^N_{NS,i}(\frac{Q^2}{\mu^2},\alpha_s) \quad &= \quad 0 \\
\sum_k \left[\left(\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\right)\delta_{jk} - \gamma^N_{jk}(\alpha_s)\right] C^N_{i,k}(\frac{Q^2}{\mu^2},\alpha_s) \quad &= \quad 0\,, \qquad j,k=q,g
\end{aligned} \tag{3.14}$$

The expansion of the β- and γ-function in α_s can be calculated from the renormalization constants by looking at the coefficient of the simple poles in the dimensional regularization parameter ϵ, $d=4-\epsilon$

$$\beta = \frac{1}{2}\alpha_s\frac{\partial}{\partial\alpha_s}Z^{(1)}_{\alpha_s}\,, \quad \gamma^N_{NS} = -\alpha_s\frac{\partial}{\partial\alpha_s}Z^{(1),N}_{NS}(\alpha_s)\,, \quad \gamma^N_{ij} = -\alpha_s\frac{\partial}{\partial\alpha_s}Z^{(1),N}_{ij}(\alpha_s) \tag{3.15}$$

where typicaly

$$Z(\alpha_s) = 1 + \frac{Z^{(1)}(\alpha_s)}{\epsilon} + \frac{Z^{(2)}(\alpha_s)}{\epsilon^2} + \cdots \tag{3.16}$$

4 The two loop anomalous dimensions of Wilson operators

The anomalous dimensions of Wilson operators O^N_i at one and two loop level for any representation of fermions are known for the following cases. For $\mathcal{N}=0$ the anomalous dimension for QCD were calculated firstly at one loop in [22, 23] and at two loop levels in [24]. It has been recalculated in [25] and a discrepancy was found for the two loop γ_{gg} in the coefficient of the Casimir C_A^2. In [19] through the supersymmetric identities that will be discussed below the discrepancy was resolved in favor of [25]. This coefficient has been recalculated in [26] and the result was found to be in agreement with that of [25]. The complete two loop QCD results were recalculated in [27, 28]. In [29], the two-loop anomalous dimensions were written in a very compact form and for phenomenological reasons the large spin behavior was studied and the asymptotic $logJ$ behavior was singled out.

The $\mathcal{N}=1$ case was studied in one-loop by [30] where the supersymmetric identity for the singlet anomalous dimensions was found

$$\gamma_{qq}^{(0)}(J)+\gamma_{gq}^{(0)}(J)=\gamma_{qg}^{(0)}(J)+\gamma_{gg}^{(0)}(J) \tag{4.1}$$

for every J. This infinite number of relations, is due to the fact that the combination of the Wilson operators $O_q^J+O_g^J$ is multiplicatively renormalized [19, 30]. For higher supersymmetries $\mathcal{N}=2$, $\mathcal{N}=4$ there are similar relations involving the scalar operator anomalous dimensions [31, 32]. Note that in the non-supersymmetric case, eq.(4.1) holds only for $J=2$ (energy-momentum tensor conservation). At the two-loop order one can check that the anomalous dimension singlet matrix elements obtained above satisfy the same relations[19]. These infinite number of relations provide an important check of the calculation and at the same time they confirm that the dimensional reduction scheme(DR) is the appropriate one for supersymmetric gauge theories[33].

The $\mathcal{N}=1$ case with quark multiplets (SUSY-QCD) was studied for phenomenological reasons at the two-loop level in the approximation of light gluinos and heavy squarks [34]. The contribution of heavy squarks was omitted from the two loop anomalous dimensions and the Q^2 evolution of the structure functions. As a result the $\mathcal{N}=2$ (quark hypermultiplet in the adjoint) two-loop anomalous dimensions could not be obtained, as the $\mathcal{N}=1$ is incomplete. On the other hand the complete $\mathcal{N}=2$ anomalous dimensions were obtained at one-loop level and additional supersymmetric relations for the singlet case were found in [31].

The $\mathcal{N}=4$ anomalous dimensions at one loop are also known and it is claimed that the two loop result can be obtained from the analytic properties of the DGLAP and BFKL evolution kernel [32].

In the following we review the results of [19] and proceed to obtain the explicit form of the anomalous dimensions for the $N=1$ supersymmetric case at the two loop level from the already known results in QCD. To this end we put the fermions in the adjoint representation and consider only the ones of the Majorana type. This can be obtained directly from the non-susy results for the special case of $C_F=C_A=2T_R=N$. However, beyond one-loop, the dimensional renormalization scheme $\bar{\mathrm{MS}}$, in which the QCD results mentioned above are obtained, breaks supersymmetry as well as does the covariant gauge fixing. The latter is solved by calculating anomalous dimensions of gauge invariant observables like the Wilson operators. In order to preserve supersymmetry, we have to use the dimensional reduction scheme (DR) in which the momentum integrations are done in $4-\epsilon$ dimensions and the spin-index algebra is performed in 4 dimensions. Instead of repeating the long two-loop calculations, there is a way to pass from $\bar{\mathrm{MS}}$ to DR if we calculate the one-loop finite part of Wilson operators in both schemes and the relation between the $\mathcal{N}=1$ YM gauge couplings between the two schemes at two loops

$$\alpha_{sDR}=\alpha_{s\bar{M}S}+\frac{1}{3}\alpha_{s\bar{M}S}^{2}+... \tag{4.2}$$

For the singlet anomalous dimensions, the relevant transformation rule between the two schemes at two-loops is [19]

$$\gamma^{(1)}_{DR} + b_0 O^{(0)}_{DR} + [\gamma^{(0)}_{DR}, O^{(0)}_{DR}] = \gamma^{(1)}_{\bar{M}S} + b_0 O^{(0)}_{\bar{M}S} + [\gamma^{(0)}_{\bar{M}S}, O^{(0)}_{\bar{M}S}] - \frac{1}{3}\gamma^{(0)}_{\bar{M}S} \tag{4.3}$$

where the two-by-two matrix of the finite parts of Wilson operators is defined as

$$O = \begin{pmatrix} O_{qq} & O_{qg} \\ O_{gq} & O_{gg} \end{pmatrix} \tag{4.4}$$

$$O = \alpha_s O^{(0)} + \alpha_s^2 O^{(1)} + \ldots \tag{4.5}$$

By employing eq.(4.3) and the two loop results from the Appendix we find the $\mathcal{N} = 1$ supersymmetric singlet anomalous dimensions. In the following we will omit the overall factor C_A^2 as well as the $-\frac{1}{3}\gamma^{(0)}_{\overline{\rm MS}}$ contribution. The $\gamma^{(1)}$'s which are portrayed below are calculated in the DR scheme

$$\begin{aligned}
\gamma^{(1)}_{qq}(J) \;=\;& -14 + \frac{8\,\left(2+J+J^2\right)}{J\,(1+J)^2\,(2+J)} - \frac{4\,\left(18+39\,J+142\,J^2+290\,J^3+151\,J^4\right)}{9\,J^3\,(1+J)^3} \\
& - \frac{8\,\left(8+28\,J+38\,J^2+49\,J^3+32\,J^4+5\,J^5\right)}{(-1+J)\,J^3\,(1+J)^3\,(2+J)^2} + \left(\frac{2}{J\,(1+J)}\right) \\
& + \frac{8\,\left(-3+11\,J^2+18 S_1(J)+J(5+36 S_1(J))\right)}{9 J^2\,(1+J)^2} + 8\left(\frac{1}{J\,(1+J)} - 2\,S_1(N)\right) S_2'(J) \\
& + \frac{152\,S_1(J)}{3} + 32\,\tilde{S}(J) - 4\,S_3'(J)
\end{aligned}$$

$$\begin{aligned}
\gamma^{(1s)}_{qg}(J) \;=\;& \frac{4\,(2+J+J^2)}{J\,(1+J)\,(2+J)}\left(-\frac{10}{3} - \frac{2}{J} + \frac{6}{1+J} - \frac{4}{2+J} + 2\,S_2'(J)\right) \\
& - \frac{4\,(4+8\,J+15\,J^2+26\,J^3+11\,J^4)}{J^3\,(1+J)^3\,(2+J)} + \frac{16\,S_1(J)}{J^2} - \frac{32\,(3+2\,J)\,S_1(J)}{(1+J)^2\,(2+J)^2} \\
& - \frac{8\,\left(16+64\,J+104\,J^2+128\,J^3+85\,J^4+36\,J^5+25\,J^6+15\,J^7+6\,J^8+J^9\right)}{(-1+J)\,J^3\,(1+J)^3\,(2+J)^3}
\end{aligned}$$

$$\begin{aligned}
\gamma^{(1)}_{gq}(J) \;=\;& -\frac{8\,\left(2+J+J^2\right)\,\left(-16+51\,S_1(J)-9\,S_2'(J)\right)}{9\,J\,\left(-1+J^2\right)} + \frac{16\,\left(-1+3\,S_1(J)\right)}{3\,(1+J)^2} \\
& + \frac{4\,\left(-4-12\,J-J^2+28\,J^3+43\,J^4+30\,J^5+12\,J^6\right)}{(-1+J)\,J^3\,(1+J)^3} \\
& - \frac{8\,\left(144+432J-152J^2-1304J^3-1031J^4+695J^5+1678J^6+1400J^7+621J^8+109J^9\right)}{9\,J^3\,(1+J)^3\,(-2+J+J^2)^2} \\
& + \frac{8\,\left(-12-22\,J+41\,J^2+17\,J^4\right)\,S_1(J)}{3\,(-1+J)^2\,J^2\,(1+J)}
\end{aligned}$$

$$
\begin{aligned}
\gamma_{gg}^{(1)}(J) \quad = \quad & \frac{4\left(-576-1488J-560J^2+1632J^3+2344J^4-1567J^5-6098J^6-6040J^7-2742J^8-457J^9\right)}{9\left(-1+J\right)^2 J^3\left(1+J\right)^3\left(2+J\right)^3} \\
& + \frac{16\ \left(-1+J+J^2\right)}{J\left(1+J\right)^2\ \left(2+J\right)} + \frac{8\ \left(-4-4J-5\,J^2-10\,J^3+J^4+4\,J^5+2\,J^6\right)}{\left(-1+J\right)\ J^3\left(1+J\right)^3\ \left(2+J\right)} + \\
& + \frac{8\ \left(12+56\,J+94\,J^2+76\,J^3+38\,J^4\right)}{9\ \left(-1+J\right)\ J^2\left(1+J\right)^2\ \left(2+J\right)} + \frac{32\ \left(1+J+J^2\right)\ S_2'(J)}{\left(-1+J\right)\ J\ \left(1+J\right)\ \left(2+J\right)} \\
& + \frac{64\ \left(-2-2\,J+7\,J^2+8\,J^3+5\,J^4+2\,J^5\right)\ S_1(J)}{\left(-1+J\right)^2 J^2\left(1+J\right)^2\left(2+J\right)^2} \\
& -14 + \frac{456\,S_1(J)}{9} - 16\,S_1(J)\,S_2'(J) + 32\,\tilde{S}(J) - 4\,S_3'(J)) \qquad (4.6)
\end{aligned}
$$

5 The large spin behaviour of anomalous dimensions

In this section we discuss the large spin behavior of the anomalous dimensions of Wilson operators for supersymmetric and non-supersymmetric Yang-Mills theories. This question is relevant for the gauge theory-string duality as we have discussed above. It is also of phenomenological interest as the scaling violations become more prominent in the kinematic regime of $x_B \to 1$ [22, 29]. The Feynman rules for the Wilson operators dictate that the diagrams with gluon lines coming out from the operator vertex will contribute terms which behave like $\lambda^k\ (\log J)^{2k-1}$ [22] and sub-dominant terms at k-loop order.

In the nonsupersymmetric case the asymptotic over all $logJ$ behavior comes about after successive miraculous cancellations of all the $(logJ)^2$ and $(logJ)^3$ terms at two loops inside the gauge invariant classes of diagrams.

This cancelation is a consequence of Ward indentities for the gluon-quark-quark vertex with an insertion of the quark operator O_q^N. As an illustration the leading behaviour of the quark-quark diagramms of Fig.1 is (we do not include explicitly the $\log J$ terms of these diagrams)

$$
\begin{aligned}
&\text{diag. A}: && 4(\log J)^2 C_F^2 \\
&\text{diag. B}: && -\frac{2}{3}(\log J)^3 C_F C_A \\
&\text{diag. C}: && -4(\log J)^2(C_F^2 - \frac{1}{2}C_F C_A) \\
&\text{diag. D}: && \left(-\frac{2}{3}(\log J)^3 + 2(\log J)^2\right) C_F C_A \\
&\text{diag. E}: && \left(\frac{4}{3}(\log J)^3 - 4(\log J)^2\right) C_F C_A \qquad (5.1)
\end{aligned}
$$

At this point it is important to observe that supersymmetry ($C_F = C_A$) does not improve the $\log J$ behavior of the above diagrams. From the previous section on the other hand there appears some cancellations in the subleading terms of γ_{qg}^{1s}(especially those which behave like $(\log J)^2/J$ for large J).

We exhibit below the large J behaviour and large N of both the nonsupersymmetric and suppersymmetric anomalous dimensions at one and two loops. We absorb the N factors of the

Casimirs in the 't Hooft coupling. In this limit we typically expand the anomalous dimensions as follows:

$$\gamma^J(\lambda) = \frac{\lambda}{4\pi}\gamma^{(0)}(J) + (\frac{\lambda}{4\pi})^2\gamma^{(1)}(J) \tag{5.2}$$

In particular the non-supersymmetric asymptotic behaviour ($C_A = N, C_F = \frac{N^2-1}{2N}$ and $T_R = \frac{n_F}{2}$ for large N gives ($C_A = 2C_F = N, T_R = 0$), is for one and two loops, respectively:

$$\begin{aligned}
\gamma_{qq}^{(0)}(J) &\sim 4\log J + 4\gamma - 3 \\
\gamma_{gg}^{(0)}(J) &\sim 2\gamma_{qq}^{(0)}(J) \\
\gamma_{qq}^{(1)}(J) &\sim \frac{4}{9}(67 - 3\pi^2)\log J - \frac{1}{36}(129 + 52\pi^2 + 16\gamma(3\pi^2 - 67)) \\
\gamma_{gg}^{(1)}(J) &\sim 2\gamma_{qq}(J)
\end{aligned} \tag{5.3}$$

where γ is the Euler-Masceroni constant.

In the $\mathcal{N} = 1$ supersymmetric case ($C_A = C_F = 2T_R = N$) the asymptotic behaviour in the DR scheme is given by:

$$\begin{aligned}
\gamma_{qq}^{(0)}(J) &\sim 8\log J + 8\gamma - 6 \\
\gamma_{gg}^{(0)}(J) &\sim \gamma_{qq}^{(0)}(J) \\
\gamma_{qq}^{(1)}(J) &\sim \frac{8}{3}(19 - \pi^2)\log J - \frac{2}{3}(21 + 36\zeta(3) + 4\gamma(\pi^2 - 19)) \\
\gamma_{gg}^{(1)}(J) &\sim \gamma_{qq}^{(1)}(J)
\end{aligned} \tag{5.4}$$

At this point we observe that by passing in the supersymmetric case from the $\bar{M}S$ to the DR scheme one must include the one-loop finite part of Wilson operators. These matrix elements contain $(\log J)^2$ terms for large J but their coefficients are the same in the two-schemes. From the relation eq.(4.3) between the two schemes only the difference of the matrix elements for the two schemes participates and so $(\log J)^2$ terms cancel.

As far as the off-diagonal elements is concerned the one- and two-loop singlet anomalous dimensions tend to zero in both the supersymmetric and non-supersymmetric case. At this point we remark that there is important piece of literature for the resummation methods of the leading behaviour at one and two loops of the structure functions near $x_B \to 1$ [32]. We would like to draw attention to the study of this limit through the cusp singularities of the Wilson loop[35]

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Appendix

$$\gamma_{NS}(J) = C_F^2 \left(\frac{16 S_1(J)(2J+1)}{J^2(J+1)^2} + 16\Big(2S_1(J) - \frac{1}{J(J+1)}\Big)(S_2(J) - S_2'(J)) + 24 S_2(J) + 64\tilde{S}(J) - 8S_3'(J) - 3 - \frac{8(1+4J+5J^2+3J^3)}{J^3(J+1)^3} \right) + C_A C_F \left(\frac{536}{9} S_1(J) - 8\Big(2S_1(J) - \frac{1}{J(J+1)}\Big)(2S_2(J) - S_2'(J)) - \frac{88}{3} S_2(J) - 28\tilde{S}(J) - \frac{17}{3} - \frac{4}{9}\frac{(-33+52J+236J^2+151J^3)}{J^2(J+1)^3} \right) + C_F T_R \left[-\frac{160}{9} S_1(J) + \frac{32}{3} S_2(J) + \frac{4}{3} + \frac{16}{9}\frac{(11J^2+5J-3)}{J^2(J+1)^2} \right] \tag{A-1}$$

$$\gamma_{qq}(J) = \gamma_{NS}(J) - 16 C_F T_R \left[\frac{(5J^5+32J^4+49J^3+38J^2+28J+8)}{(J-1)J^3(J+1)^3(J+2)^2} \right] \tag{A-2}$$

$$\gamma_{qg}(J) = -8 C_F T_R \left[\frac{(4+8J+15J^2+26J^3+11J^4)}{J^3(J+1)^3(J+2)} - \frac{4S_1(J)}{J^2} + \frac{(2+J+J^2)(5+2S_1^2(J) - 2S_2(J))}{J(J+1)(J+2)} \right] - 8 C_A T_R \left[\frac{2(16+64J+104J^2+128J^3+85J^4+36J^5+25J^6+15J^7+6J^8+J^9)}{(J-1)J^3(J+1)^3(J+2)^3} + \frac{8(3+2J)S_1(J)}{(J+1)^2(J+2)^2} + \frac{(2+J+J^2)(-2S_1^2(J) + 2S_2(J) - 2S_2'(J))}{J(J+1)(J+2)} \right] \tag{A-3}$$

$$\gamma_{gq}(J) = -\frac{32}{3} C_F T_R \left[\frac{1}{(J+1)^2} + \frac{(2+J+J^2)(-8/3+S_1(J))}{(J-1)J(J+1)} \right] - 4C_F^2 \left[-\frac{(-4-12J-J^2+28J^3+43J^4+30J^5+12J^6)}{(J-1)J^3(J+1)^3} - \frac{4S_1(J)}{(J+1)^2} + \frac{(2+J+J^2)(10S_1(J) - 2S_1^2(J) - 2S_2(J))}{(J-1)J(J+1)} \right] - 8 C_A C_F \left[\frac{(144+432J-152J^2-1304J^3-1031J^4+695J^5)}{9(J-1)^2J^3(J+1)^3(J+2)^3} + \frac{(+1678J^6+1400J^7+621J^8+109J^9)}{9(J-1)^2J^3(J+1)^3(J+2)^2} \right. \tag{A-4}$$

$$
- \frac{(-12 - 22J + 41J^2 + 17J^4)S_1(J)}{3(J-1)^2 J^2 (J+1)} + \frac{(2 + J + J^2)(S_1^2(J) + S_2(J) - S_2'(J)}{(J-1)J(J+1)} \Bigg]
$$

$$
\begin{aligned}
\gamma_{gg}(J) \quad = \quad & C_F T_R \left[8 + \frac{16(-4 - 4J - 5J^2 - 10J^3 + J^4 + 4J^5 + 2J^6)}{(J-1)J^3(J+1)^3(J+2)} \right] \\
& + C_A T_R \left[\frac{32}{3} + \frac{16(12 + 56J + 94J^2 + 76J^3 + 38J^4)}{9(J-1)J^2(J+1)^2(J+2)} - \frac{160 S_1(J)}{9} \right] \\
& + C_A^2 \Bigg[- \frac{4(576 + 1488J + 560J^2 - 1632J^3 - 2344J^4 + 1567J^5)}{9(J-1)^2 J^3 (J+1)^3 (J+2)^3} \\
& + \frac{(6098J^3 + 6040J^4 + 2742J^5 + 457J^6)}{9(J-1)^2(J+1)^3(J+2)^3} \\
& - \frac{64}{3} + \frac{536}{9} S_1(J) + \frac{64(-2 - 2J + 7J^2 + 8J^3 + 5J^4 + 2J^5)S_1(J)}{(J-1)^2 J^2 (J+1)^2 (J+2)^2} \\
& + \frac{32(1 + J + J^2) S_2'(J)}{(J-1)J(J+1)J+2)} - 16 S_1(J) S_2'(J) + 32 \tilde{S}(J) - 4 S_3'(J) \Bigg]
\end{aligned} \tag{A-5}
$$

$$
\gamma_{qq}^0(J) = 2C_F \left[4S_1(J) - 3 - \frac{2}{J(J+1)} \right] \tag{A-6}
$$

$$
\gamma_{qg}^0(J) = -\frac{8T_R(J^2 + J + 1)}{J(J+1)(J+2)} \tag{A-7}
$$

$$
\gamma_{gq}(J) = -\frac{4C_F(2 + J + J^2)}{(J-1)J(J+1)} \tag{A-8}
$$

$$
\gamma_{gg}^0(J) \quad = \quad \frac{8}{3} T_R + 2C_A \left[-\frac{11}{3} - \frac{4}{J(J-1)} - \frac{4}{(J+1)(J+2)} + 4S_1(J) \right] \tag{A-9}
$$

$$
\begin{aligned}
S_n(J) = \sum_{k=1}^{J} \frac{1}{k^n} \,, \qquad S_n'(J) = 2^{n-1} \sum_{k=1}^{J} \frac{1 + (-1)^k}{k^n} \\
\tilde{S}(J) = \sum_{k}^{J} \frac{(-1)^k S_1(J)}{k^2}
\end{aligned} \tag{A-10}
$$

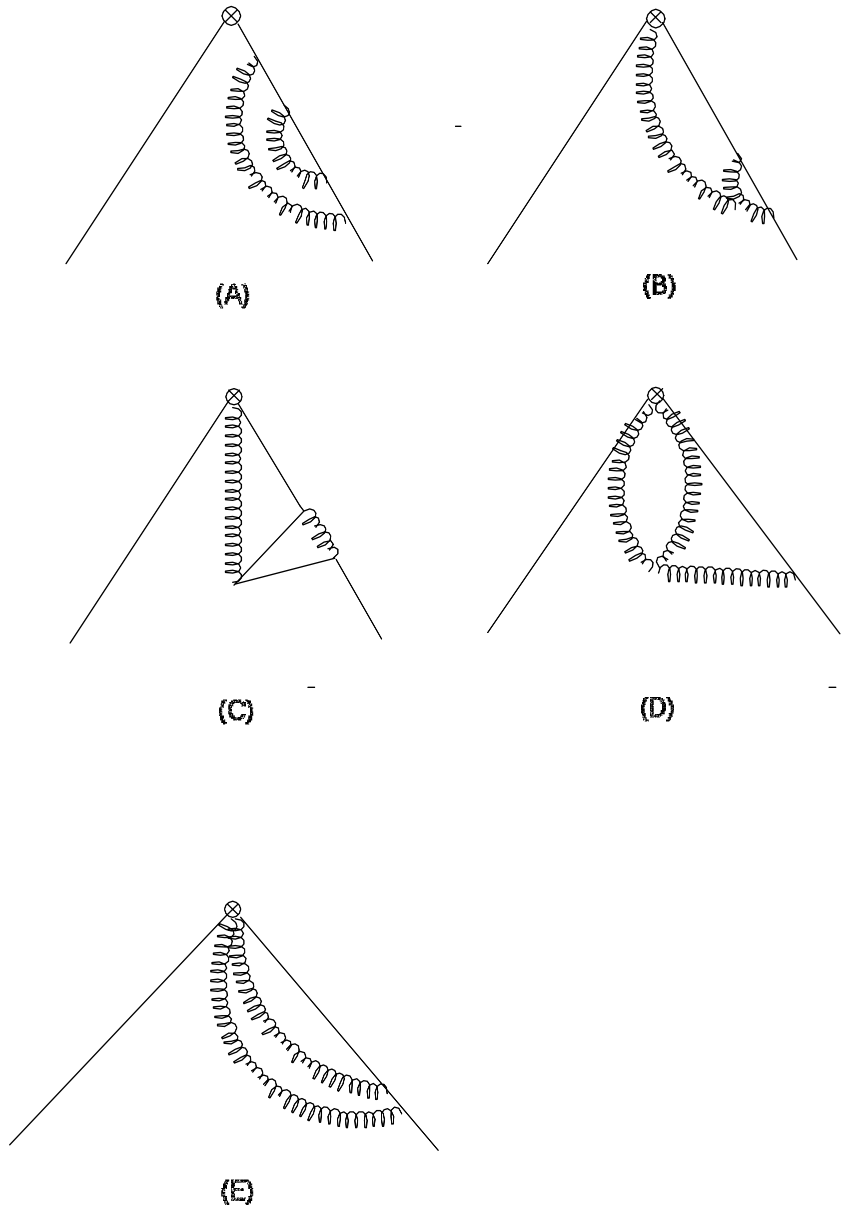

Figure 1: **Two-loop quark-quark diagramms with quadratic or cubic in $\log J$ leading behavior at large spin J**

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